2002 FP2 Adapted

1. Find the set of values for which

$$|x-1| > 6x-1. \qquad \textbf{(5)}$$

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2. (*a*) Find the general solution of the differential equation $t\dfrac{\mathrm{d}v}{\mathrm{d}t} - v = t, \quad t > 0$

and hence show that the solution can be written in the form $v = t(\ln t + c)$, where c is an arbitrary cnst. **(6)**

(*b*) This differential equation is used to model the motion of a particle which has speed v m s^{-1} at time t s. When $t = 2$ the speed of the particle is 3 m s^{-1}. Find, to 3 sf, the speed of the particle when $t = 4$. **(4)**

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3. (*a*) Show that $y = \frac{1}{2}x^2e^x$ is a solution of the differential equation

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} + y = e^x. \qquad \textbf{(4)}$$

(*b*) Solve the differential equation $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\dfrac{\mathrm{d}y}{\mathrm{d}x} + y = e^x$.

given that at $x = 0$, $y = 1$ and $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 2$. **(9)**

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4. The curve C has polar equation $r = 3a\cos\theta, -\dfrac{\pi}{2} \leq \dfrac{\pi}{2}$.

The curve D has polar equation $r = a(1 + \cos\theta), -\pi \leq \theta < \pi$. Given that a is a positive constant,

(*a*) sketch, on the same diagram, the graphs of C and D, indicating where each curve cuts the initial line. **(4)**

The graphs of C intersect at the pole O and at the points P and Q.

(*b*) Find the polar coordinates of P and Q. **(3)**

(*c*) Use integration to find the exact area enclosed by the curve D and the lines $\theta = 0$ and $\theta = \dfrac{\pi}{3}$ **(7)**

The region R contains all points which lie outside D and inside C.

Given that the value of the smaller area enclosed by the curve C and the line $\theta = \dfrac{\pi}{3}$ is

$$\frac{3a^2}{16}(2\pi - 3\sqrt{3}),$$

(*d*) show that the area of R is πa^2. **(4)**

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5. Using algebra, find the set of values of x for which $2x - 5 > \dfrac{3}{x}$. **(7)**

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6. (*a*) Find the general solution of the differential equation

$$\cos x \frac{dy}{dx} + (\sin x)y = \cos^3 x. \quad \textbf{(6)}$$

(*b*) Show that, for $0 \le x \le 2\pi$, there are two points on the x-axis through which all the solution curves for this differential equation pass. **(2)**

(*c*) Sketch the graph, for $0 \le x \le 2\pi$, of the particular solution for which $y = 0$ at $x = 0$. **(3)**

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7. (*a*) Find the general solution of the differential equation

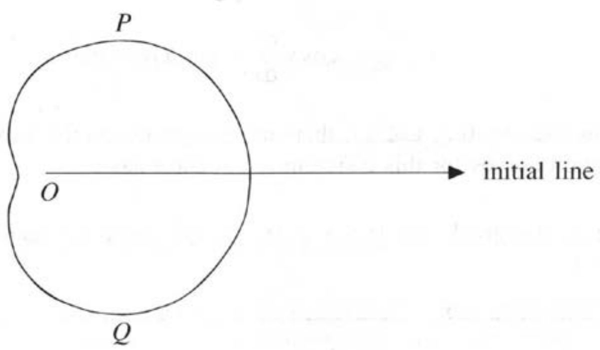
$$2\frac{d^2y}{dt^2} + 7\frac{dy}{dt} + 3y = 3t^2 + 11t. \quad \textbf{(8)}$$

(*b*) Find the particular solution of this differential equation for which $y = 1$ and $\dfrac{dy}{dt} = 1$ when $t = 0$. **(5)**

(*c*) For this particular solution, calculate the value of y when $t = 1$. **(1)**

8.

Figure 1



The curve C shown in Fig. 1 has polar equation

$$r = a(3 + \sqrt{5}\cos\theta), \quad -\pi \le \theta < \pi.$$

(*a*) Find the polar coordinates of the points P and Q where the tangents to C are parallel to the initial line. **(6)**

The curve C represents the perimeter of the surface of a swimming pool. The direct distance from P to Q is 20 m.

(*b*) Calculate the value of a. **(3)**

(*c*) Find the area of the surface of the pool. **(6)**

9. (*a*) The point P represents a complex number z in an Argand diagram. Given that

$$|z - 2\mathrm{i}| = 2|z + \mathrm{i}|,$$

(i) find a cartesian equation for the locus of P, simplifying your answer. **(2)**

(ii) sketch the locus of P. **(3)**

(*b*) A transformation T from the z-plane to the w-plane is a translation $-7 + 11\mathrm{i}$ followed by an enlargement with centre the origin and scale factor 3.

Write down the transformation T in the form

$$w = az + b, \quad a, b \in \mathbb{C}.$$ **(2)**

10.

$$y\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2 + y = 0.$$

(*a*) Find an expression for $\frac{\mathrm{d}^3y}{\mathrm{d}x^3}$. **(5)**

Given that $y = 1$ and $\frac{\mathrm{d}y}{\mathrm{d}x} = 1$ at $x = 0$,

(*b*) find the series solution for y, in ascending powers of x, up to an including the term in x^3. **(5)**

(*c*) Comment on whether it would be sensible to use your series solution to give estimates for y at $x = 0.2$ and at $x = 50$. **(2)**

Total 112 marks